Functions and Graphs – Answers

June 2017 Mathematics Advanced Paper 1: Pure Mathematics 3

1.

Question Number	Scheme	Marks
3.(a)	$y \geqslant 3$	B1
		(1)
(b)	$y = 3 + \sqrt{x+2} \Rightarrow y - 3 = \sqrt{x+2} \Rightarrow x = (y-3)^2 - 2$	M1 A1
	$\Rightarrow g^{-1}(x) = (x-3)^2 - 2, \quad \text{with } x \geqslant 3$	A1
		(3)
(c)	$g(x) = x \Rightarrow 3 + \sqrt{x+2} = x$	
	$\Rightarrow x + 2 = (x-3)^2 \Rightarrow x^2 - 7x + 7 = 0$	M1, A1
	$\Rightarrow x = \frac{7 \pm \sqrt{21}}{2} \Rightarrow x = \frac{7 + \sqrt{21}}{2}$ only	M1, A1
		(4)
(d)	$a = \frac{7 + \sqrt{21}}{2}$	B1 ft
		(1)
		9 marks
(c) Alt	Solves $g^{-1}(x) = x \Rightarrow (x-3)^2 - 2 = x$	
	$\Rightarrow x^2 - 7x + 7 = 0$	M1, A1
	$\Rightarrow x = \frac{7 \pm \sqrt{21}}{2} \Rightarrow x = \frac{7 + \sqrt{21}}{2}$ only	dM1, A1
		(4)

(a)

B1 States the correct range for g Accept $g(x)..3 3 g..3$, Range$..3$, $[3,\infty)$ Range is greater than or equal to 3

Condone $f..3$ Do not accept $g(x) > 3, x..3, (3,\infty)$

(b)

M1 Attempts to make x or a swapped y the subject of the formula. The minimum expectation is that the 3 is moved over followed by an attempt to square both sides. Condone for this mark $\sqrt{x+2} = y \pm 3 \Rightarrow x+2 = y^2 \pm 9$

A1 Achieves $x = (y-3)^2 - 2$ or if swapped $y = (x-3)^2 - 2$ or equivalent such as $x = y^2 - 6y + 7$

A1 Requires a correct function in x + correct domain **or** a correct function in x with a correct follow through on the range in (a) but do not follow through on $x \in \mathrm{R}$

Accept for example $g^{-1}(x) = (x-3)^2 - 2, \quad x..3$ Condone $f^{-1}(x) = (x-3)^2 - 2, \quad x..3$

or variations such as $y = (x-3)^2 - 2, \quad x > 3$ if (a) was $y > 3$

Accept expanded versions such as $g^{-1}(x) = x^2 - 6x + 7, \quad x..3$ but remember to isw after a correct answer

(Condone $f^{-1}(x) = x^2 - 6x + 7, \quad x..3$)

(c)

M1 Sets $3 + \sqrt{x+2} = x$, moves the 3 over and then attempts to square both sides.

Can be scored for $\sqrt{x+2} = x - 3 \Rightarrow x + 2 = x^2 \pm 9$

A1 $x^2 - 7x + 7 = 0$. The = 0 may be implied by subsequent working

M1 Correct method of solving their 3TQ by the formula/ completing the square. The equation must have real roots.

It is dependent upon them having attempted to set $3 + \sqrt{x+2} = x$ and proceeding to a quadratic.

You may just see both roots written down which is fine.

Allow for this mark decimal answers Eg 5.79 and 1.21 for $x^2 - 7x + 7 = 0$ You may need to check with a calc.

A1 $(x) = \dfrac{7+\sqrt{21}}{2}$ or exact equivalent **only**.

This answer following the correct quadratic would imply the previous M

Allow $x = \dfrac{7}{2} + \sqrt{\dfrac{21}{4}}$ but **DO NOT** allow $x = \dfrac{7 \pm \sqrt{21}}{2}$

(c) can of course be attempted by solving
$$3 + \sqrt{x+2} = \text{"}(x-3)^2 - 2\text{"} \Rightarrow x^4 - 12x^3 + 44x^2 - 49x + 14 = 0$$
$$\vdots \qquad \Rightarrow (x^2 - 7x + 7)(x^2 - 5x + 2) = 0$$

The scheme can be applied to this

....

(d)

B1ft $(a) = \dfrac{7+\sqrt{21}}{2}$ oe. You may condone $\boldsymbol{x} = \dfrac{7+\sqrt{21}}{2}$. You may allow this following a re - start.

You may allow the correct decimal answer, awrt 5.79, following exact/decimal work in part (c) or a restart.

Follow through on their root, including decimals, coming from the **positive** root with the **positive** sign in (c).

Eg In (c). $x^2 - 7x + 11 = 0 \Rightarrow x = \dfrac{7 \pm \sqrt{5}}{2}$ So the correct follow through would be $x = \dfrac{7+\sqrt{5}}{2}$

If they only had one root in (c) then follow through on this as long as it is positive.

SC. If they give the correct roots in parts (c) and (d) without considering the correct answer then award B1 in (d) following the A0 in (c). So $(x) = \dfrac{7 \pm \sqrt{21}}{2}$ as their answer in part (c), allow $(x/a) = \dfrac{7 \pm \sqrt{21}}{2}$ for B1 in (d).

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2.

Question	Scheme	Marks
1(a)	$\mathrm{fg}(x)=\dfrac{28}{x-2}-1 \qquad \left(=\dfrac{30-x}{x-2}\right)$	M1
	Sets $\mathrm{fg}(x)=x \Rightarrow \dfrac{28}{x-2}-1=x$	
	$\Rightarrow 28=(x+1)(x-2)$	
	$\Rightarrow x^2-x-30=0$	M1
	$\Rightarrow (x-6)(x+5)=0$	
	$\Rightarrow x=6, x=-5$	dM1 A1
		(4)
(b)	$a=6$	B1 ft
		(1)
		5 marks
Alt 1(a)	$\mathrm{fg}(x)=x \Rightarrow \mathrm{g}(x)=\mathrm{f}^{-1}(x)$	M1
	$\dfrac{4}{x-2}=\dfrac{x+1}{7}$	
	$\Rightarrow x^2-x-30=0$	M1
	$\Rightarrow (x-6)(x+5)=0$	
	$\Rightarrow x=6, x=-5$	dM1 A1
		4 marks

S. Case	Uses $\mathrm{gf}(x)$ instead $\mathrm{fg}(x)$	Makes an error on $\mathrm{fg}(x)$	
	$\dfrac{4}{7x-1-2}=x$	Sets $\mathrm{fg}(x)=x \Rightarrow \dfrac{7\times 4}{7\times(x-2)}-1=x$	M0
	$\Rightarrow 7x^2-3x-4=0$	$\Rightarrow x^2-x-6=0$	M1
	$\Rightarrow (7x+4)(x-1)=0$	$\Rightarrow (x+2)(x-3)=0$	
	$\Rightarrow x=-\dfrac{4}{7}, \quad x=1$	$\Rightarrow x=-2, \quad x=3$	dM1 A0
			2 out of 4 marks

(a)

M1 Sets or implies that $\mathrm{fg}(x)=\frac{28}{x-2}-1$ Eg accept $\mathrm{fg}(x)=7\left(\frac{4}{x-2}\right)-1$ followed by $\mathrm{fg}(x)=\frac{7\times4}{x-2}-1$

Alternatively sets $\mathrm{g}(x)=\mathrm{f}^{-1}(x)$ where $\mathrm{f}^{-1}(x)=\frac{x\pm1}{7}$

Note that $\mathrm{fg}(x)=7\left(\frac{4}{x-2}\right)-1=\frac{28}{7(x-2)}-1$ is M0

M1 Sets up a 3TQ (= 0) from an attempt at $\mathrm{fg}(x)=x$ or $\mathrm{g}(x)=\mathrm{f}^{-1}(x)$

dM1 Method of solving 3TQ (= 0) to find at least one value for x. See "General Priciples for Core Mathematics" on page 3 for the award of the mark for solving quadratic equations
This is dependent upon the previous M. You may just see the answers following the 3TQ.

A1 Both $x=6$ and $x=-5$

(b)

B1ft For $a=6$ but you may follow through on the largest solution from part (a) provided more than one answer was found in (a). Accept 6, $a=6$ and even $x=6$
Do not award marks for part (a) for work in part (b).

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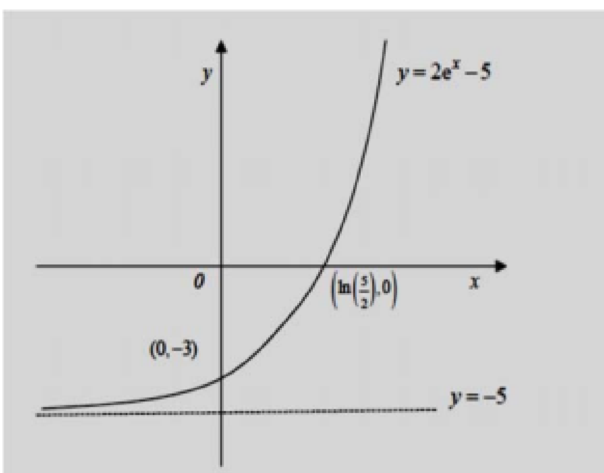
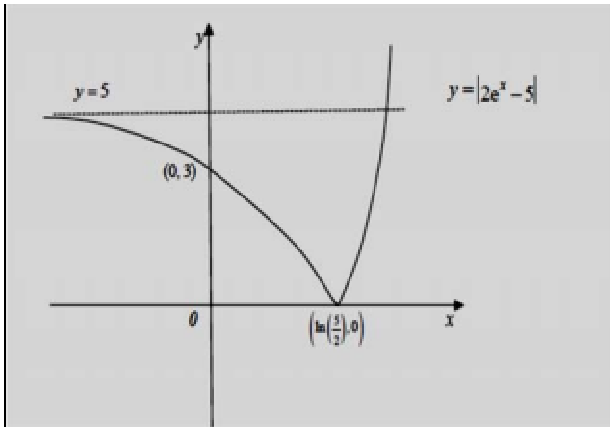
3.

Question Number	Scheme		Marks
2.(ai)		Shape	B1
		$\left(\ln\left(\frac{5}{2}\right),0\right)$ and $(0,-3)$	B1
		$y=-5$	B1
			(3)

(aii)	 Shape inc cusp $\left(\ln\left(\frac{5}{2}\right),0\right)$ *and* $(0,3)$ $y=5$	B1ft B1ft B1ft **(3)**
(b)	$x \geqslant \ln\left(\frac{5}{2}\right)$	B1 ft **(1)**
(c)	$2e^x - 5 = -2 \Rightarrow (x) = \ln\left(\frac{3}{2}\right)$ $(x) = \ln\left(\frac{7}{2}\right)$	M1A1 B1 **(3)** **(10 marks)**

(a)(i)

B1 For an exponential (growth) shaped curve in any position. For this mark be tolerant on slips of the pen at either end. See Practice and Qualification for examples.

B1 Intersections with the axes at $\left(\ln\left(\frac{5}{2}\right),0\right)$ and $(0,-3)$.

Allow $\ln\left(\frac{5}{2}\right)$ and -3 being marked on the correct axes.

Condone $\left(0,\ln\left(\frac{5}{2}\right)\right)$ and $(-3,0)$ being marked on the x and y axes respectively.

Do not allow $\left(\ln\left(\frac{5}{2}\right),0\right)$ appearing as awrt (0.92, 0) for this mark unless seen elsewhere. Allow if seen in body of script. If they are given in the body of the script and differently on the curve (save for the decimal equivalent) then **the ones on the curve take precedence.**

B1 Equation of the asymptote given as $y=-5$. Note that the curve must appear to have an asymptote at $y=-5$, not necessarily drawn. It is not enough to have -5 marked on the axis or indeed $x=-5$. An extra asymptote **with an equation** gets B0 (a)(ii)

B1ft For **either** the correct shape **or** a reflection of their curve from (a)(i) in the x- axis. For this to be scored it must have appeared both above and below the x - axis. The shape must be correct including the cusp. The curve to the lhs of the cusp must appear to have the correct curvature

B1ft Score for both intersections or follow through on both the intersections given in part (a)(i), including decimals, as long as the curve appeared both above and below the x - axis. See part (a) for acceptable forms

B1ft Score for an asymptote of $y = 5$ or follow through on an asymptote of $y = -C$ from part (a)(i). Note that the curve must appear to have an asymptote at $y = C$ but do not penalise if the first mark in (a)(ii) has been withheld for incorrect curvature on the lhs.

(b)

B1ft Score for $x \geqslant \ln\left(\frac{5}{2}\right)$, $x \geqslant$ awrt 0.92 or follow through on the x intersection in part (a)

(c)

M1 Accept $2e^x - 5 = -2$ or $-2e^x + 5 = 2 \Rightarrow x = ..\ln(..)$

Allow squaring so $(2e^x - 5)^2 = 4 \Rightarrow e^x = ..$ and $.. \Rightarrow x = \ln(..), \ln(..)$

A1 $x = \ln\left(\frac{3}{2}\right)$ or exact equivalents such as $x = \ln 1.5$. You do not need to see the x.

Remember to isw a subsequent decimal answer 0.405

B1 $x = \ln\left(\frac{7}{2}\right)$ or exact equivalents such as $x = \ln 3.5$. You do not need to see the x.

Remember to isw a subsequent decimal answer 1.25

If both answers are given in decimals and there is no working $x =$ awrt 1.25, 0.405 award SC 100

4.

Question Number	Scheme	Marks
7.(a)	Applies $vu'+uv'$ to $(x^2-x^3)e^{-2x}$	
	$g'(x)=(x^2-x^3)\times -2e^{-2x}+(2x-3x^2)\times e^{-2x}$	M1 A1
	$g'(x)=(2x^3-5x^2+2x)e^{-2x}$	A1
		(3)
(b)	Sets $(2x^3-5x^2+2x)e^{-2x}=0 \Rightarrow 2x^3-5x^2+2x=0$	M1
	$x(2x^2-5x+2)=0 \Rightarrow x=(0),\frac{1}{2},2$	M1,A1
	Sub $x=\frac{1}{2}$, 2 into $g(x)=(x^2-x^3)e^{-2x} \Rightarrow g\left(\frac{1}{2}\right)=\frac{1}{8e},\quad g(2)=-\frac{4}{e^4}$	dM1,A1
	Range $-\frac{4}{e^4}\leqslant g(x)\leqslant\frac{1}{8e}$	A1 **(6)**
(c)	Accept $g(x)$ is NOT a ONE to ONE function	
	Accept $g(x)$ is a MANY to ONE function	B1
	Accept $g^{-1}(x)$ would be ONE to MANY	**(1)**
		(10 marks)

Note that parts (a) and (b) can be scored together. Eg accept work in part (b) for part (a)

(a)

M1 **Uses the product rule** $vu'+uv'$ with $u=x^2-x^3$ and $v=e^{-2x}$ or vice versa. If the rule is quoted it must be correct. It may be implied by their $u=..v=..u'=..v'=..$ followed by their $vu'+uv'$. If the rule is not quoted nor implied only accept expressions of the form $(x^2-x^3)\times\pm Ae^{-2x}+(Bx\pm Cx^2)\times e^{-2x}$ condoning bracketing issues

Method 2: multiplies out and **uses the product rule** on each term of $x^2e^{-2x}-x^3e^{-2x}$
Condone issues in the signs of the last two terms for the method mark
Uses the product rule for $uvw=u'vw+uv'w+uvw'$ applied as in method 1

Method 3:Uses **the quotient rule** with $u=x^2-x^3$ and $v=e^{2x}$. If the rule is quoted it must be correct. It may be implied by their $u=..v=..u'=..v'=..$ followed by their $\frac{vu'-uv'}{v^2}$ If the rule is not quoted nor implied accept expressions of the form $\frac{e^{2x}(Ax-Bx^2)-(x^2-x^3)\times Ce^{2x}}{(e^{2x})^2}$

condoning missing brackets on the numerator and e^{2x^2} on the denominator.

Method 4: Apply implicit differentiation to $ye^{2x}=x^2-x^3 \Rightarrow e^{2x}\times\frac{dy}{dx}+y\times 2e^{2x}=2x-3x^2$

Condone errors on coefficients and signs

A1 A correct (unsimplified form) of the answer

$g'(x) = (x^2 - x^3) \times -2e^{-2x} + (2x - 3x^2) \times e^{-2x}$ by one use of the product rule

or $g'(x) = x^2 \times -2e^{-2x} + 2xe^{-2x} - x^3 \times -2e^{-2x} - 3x^2 \times e^{-2x}$ using the first alternative

or $g'(x) = 2x(1-x)e^{-2x} + x^2 \times -1 \times e^{-2x} + x^2(1-x) \times -2e^{-2x}$ using the product rule on 3 terms

or $g'(x) = \dfrac{e^{2x}(2x - 3x^2) - (x^2 - x^3) \times 2e^{2x}}{(e^{2x})^2}$ using the quotient rule.

A1 Writes $g'(x) = (2x^3 - 5x^2 + 2x)e^{-2x}$. You do not need to see $f(x)$ stated and award even if a correct $g'(x)$ is followed by an incorrect $f(x)$. If the f(x) is not simplified at this stage you need to see it simplified later for this to be awarded.

(b) Note: The last mark in e-pen has been changed from a 'B' to an A mark

M1 For setting their f(x) =0. The = 0 may be implied by subsequent working.
Allow even if the candidate has failed to reach a 3TC for f(x).
Allow for $f(x) \geqslant 0$ or $f(x) \leqslant 0$ as they can use this to pick out the relevant sections of the curve

M1 For solving their 3TC = 0 by ANY correct method.
Allow for division of x or factorising out the x followed by factorisation of 3TQ. Check first and last terms of the 3TQ. Allow for solutions from either $f(x) \geqslant 0$ or $f(x) \leqslant 0$
Allow solutions **from the cubic equation** just appearing from a Graphical Calculator

A1 $x = \frac{1}{2}, 2$. Correct answers from a correct $g'(x)$ would imply all 3 marks so far in (b)

dM1 Dependent upon both previous M's being scored. For substituting their **two** (non zero) values of x into g(x) to find both y values. Minimal evidence is required $x = .. \Rightarrow y = ..$ is OK.

A1 Accept decimal answers for this mark. $g\left(\frac{1}{2}\right) = \frac{1}{8e}$=awrt 0.046 **AND** $g(2) = -\frac{4}{e^4}$=awrt -0.073

A1 CSO Allow $-\frac{4}{e^4} \leqslant \text{Range} \leqslant \frac{1}{8e}$, $-\frac{4}{e^4} \leqslant y \leqslant \frac{1}{8e}$, $\left[-\frac{4}{e^4}, \frac{1}{8e}\right]$. Condone $y \geqslant -\frac{4}{e^4}$ $y \leqslant \frac{1}{8e}$

Note that the question states hence and part (a) must have been used for all marks. Some students will just write down the answers for the range from a graphical calculator.
Seeing just $-\frac{4}{e^4} \leqslant g(x) \leqslant \frac{1}{8e}$ or $-0.073 \leqslant g(x) \leqslant 0.046$ special case 100000.
They know what a range is!

(c)

B1 If the candidate states 'NOT ONE TO ONE' then accept unless the explicitly link it to $g^{-1}(x)$. So accept 'It is not a one to one function'. 'The function is not one to one' '$g(x)$ is not one to one'
If the candidate states 'IT IS MANY TO ONE' then accept unless the candidate explicitly links it to $g^{-1}(x)$. So accept 'It is a many to one function.' 'The function is many to one' '$g(x)$ is many to one'
If the candidate states 'IT IS ONE TO MANY' then accept unless the candidate explicitly links it to $g(x)$
Accept an explanation like " one value of x would map/ go to more than one value of y"
Incorrect statements scoring B0 would be $g^{-1}(x)$ is not one to one, $g^{-1}(x)$ is many to one and $g(x)$ is one to many.

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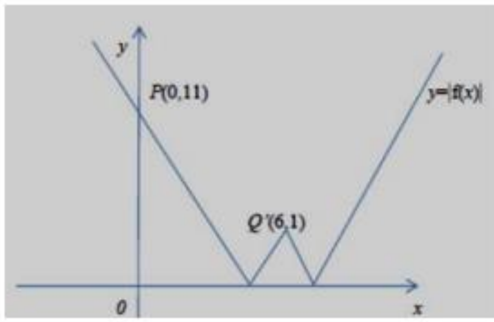
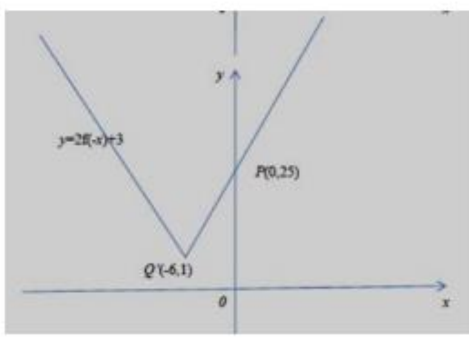
5.

Question Number	Scheme		Marks
4.(a)		'W' Shape	B1
		(0, 11) and (6, 1)	B1
			(2)
(b)		'V' shape	B1
		(-6,1)	B1
		(0,25)	B1
			(3)
(c)	One of $a = 2$ or $b = 6$		B1
	$a = 2$ and $b = 6$		B1
			(2)
			(7 marks)

(a)

B1 A W shape in any position. The arms of the W do not need to be symmetrical but the two bottom points must appear to be at the same height. Do not accept rounded W's.
A correct sketch of $y = \mathrm{f}(|x|)$ would score this mark.

B1 A W shape in quadrants 1 and 2 sitting on the x axis with $P' = (0,11)$ **and** $Q' = (6,1)$. It is not necessary to see them labelled. Accept 11 being marked on the y axis for P'. Condone $P' = (11,0)$ marked on the correct axis, but $Q' = (1,6)$ is B0

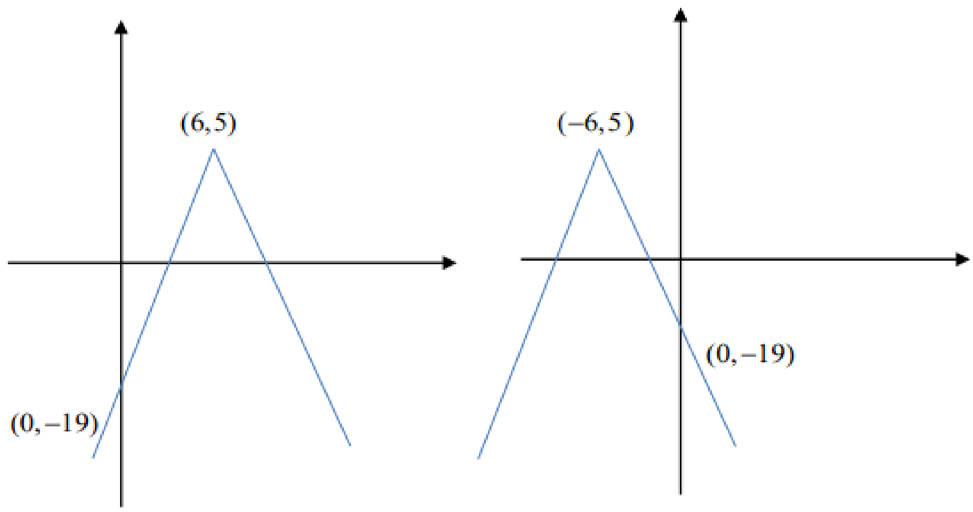
(b)

B1 Score for a V shape in any position on the grid. The arms of the V do not need to be symmetrical. Do not accept rounded or upside down V's for this mark.

B1 $Q' = (-6, 1)$. It does not need to be labelled but it must correspond to the minimum point on the curve and be in the correct quadrant.

B1 $P' = (0, 25)$. It does not need to be labelled but it must correspond to the y intercept and the line must cross the axis. Accept 25 marked on the correct axis. Condone $P' = (25, 0)$ marked on the positive y axis.

Special case: A candidate who mistakenly sketches $y = -2f(x) + 3$ or $y = -2f(-x) + 3$ will arrive at one of the following. They can be awarded SC B1B0B0



(c)

B1 Either states $a = 2$ **or** $b = 6$.
This can be implied (if there are no stated answers given) by the candidate writing that $y = ..|x-6|-1$ or $y = 2|x - ..| - 1$. If they are both stated and written, the stated answer takes precedence.

B1 States both $a = 2$ **and** $b = 6$
This can be implied by the candidate stating that $y = 2|x-6|-1$
If they are both stated and written, the stated answer takes precedence.

6.

Question Number	Scheme	Marks
5.(a)	$x^2+x-6=(x+3)(x-2)$	B1
	$\frac{x}{x+3}+\frac{3(2x+1)}{(x+3)(x-2)}=\frac{x(x-2)+3(2x+1)}{(x+3)(x-2)}$	M1
	$=\frac{x^2+4x+3}{(x+3)(x-2)}$	A1
	$=\frac{(x+3)(x+1)}{(x+3)(x-2)}$	
	$=\frac{(x+1)}{(x-2)}$ cso	A1*
		(4)
(b)	One end either $(y)>1,(y)\geqslant 1$ or $(y)<4,(y)\leqslant 4$	B1
	$1<y<4$	B1
		(2)
(c)	Attempt to set Either $g(x)=x$ or $g(x)=g^{-1}(x)$ or $g^{-1}(x)=x$ or $g^2(x)=x$ $\frac{(x+1)}{(x-2)}=x$ $\frac{x+1}{x-2}=\frac{2x+1}{x-1}$ $\frac{2x+1}{x-1}=x$ $\frac{\frac{x+1}{x-2}+1}{\frac{x+1}{x-2}-2}=x$	M1
	$x^2-3x-1=0\Rightarrow x=...$	A1, dM1
	$a=\frac{3+\sqrt{13}}{2}$ oe $\left(1.5+\sqrt{3.25}\right)$ cso	A1
		(4) (10 marks)

(a)

B1 $x^2+x-6=(x+3)(x-2)$ This can occur anywhere in the solution.

M1 For combining the two fractions with a common denominator. The denominator must be correct for their fractions and at least one numerator must have been adapted. Accept as separate fractions. Condone missing brackets.

Accept $\frac{x}{x+3}+\frac{3(2x+1)}{x^2+x-6}=\frac{x(x^2+x-6)+3(2x+1)(x+3)}{(x+3)(x^2+x-6)}$

Condone $\frac{x}{x+3}+\frac{3(2x+1)}{(x+3)(x-2)}=\frac{x\times x-2}{(x+3)(x-2)}+\frac{3(2x+1)}{(x+3)(x-2)}$

A1 A correct intermediate form of $\frac{\text{simplified quadratic}}{\text{simplified quadratic}}$

Accept $\frac{x^2+4x+3}{(x+3)(x-2)}, \frac{x^2+4x+3}{x^2+x-6}$, OR $\frac{x^3+7x^2+15x+9}{(x+3)(x^2+x-6)} \rightarrow \frac{(x+1)(x+3)\cancel{(x+3)}}{\cancel{(x+3)}(x^2+x-6)}$

As in question one they can score this mark having 'invisible' brackets on line 1.

A1* Further factorises and cancels (which may be implied) to complete the proof to reach the given answer $=\frac{(x+1)}{(x-2)}$. All aspects including bracketing must be correct. If a cubic is formed then it needs to be correct.

(b)

B1 States either end of the range. Accept either $y<4, y\leqslant 4$ or $y>1, y\geqslant 1$ with or without the y's.

B1 Correct range. Accept $1<y<4$, $1<\mathrm{g}<4$, $y>1$ and $y<4$, $(1,4)$, $1<\text{Range}<4$, even $1<\mathrm{f}<4$,

Do not accept $1<x<4$, $1<y\leq 4$, $[1,4)$ etc.

Special case, allow B1B0 for $1<x<4$

(c)

M1 Attempting to set $g(x)=x$, $g^{-1}(x)=x$ or $g(x)=g^{-1}(x)$ or $g^2(x)=x$.

If $g^{-1}(x)$ has been used then a full attempt must have been made to make x the subject of the formula. A full attempt would involve cross multiplying, collecting terms, factorising and ending with division.

As a result, it must be in the form $g^{-1}(x)=\frac{\pm 2x\pm 1}{\pm x\pm 1}$

Accept as evidence $\frac{(x+1)}{(x-2)}=x$ OR $\frac{x+1}{x-2}=\frac{\pm 2x\pm 1}{\pm x\pm 1}$ OR $\frac{\pm 2x\pm 1}{\pm x\pm 1}=x$ OR $\frac{\frac{x+1}{x-2}+1}{\frac{x+1}{x-2}-2}=x$

A1 $x^2-3x-1=0$ or exact equivalent. The =0 may be implied by subsequent work.

dM1 For solving a 3TQ=0. It is dependent upon the first M being scored. Do not accept a method using factors unless it clearly factorises. Allow the answer written down awrt 3.30 (from a graphical calculator).

A1 a or $x=\frac{3+\sqrt{13}}{2}$. Ignore any reference to $\frac{3-\sqrt{13}}{2}$

Withhold this mark if additional values are given for x, $x>3$

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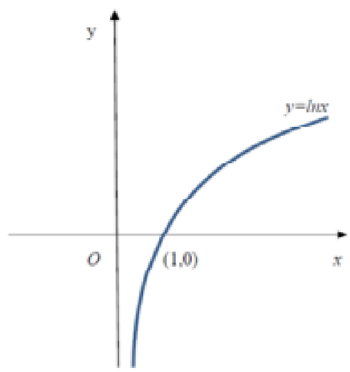
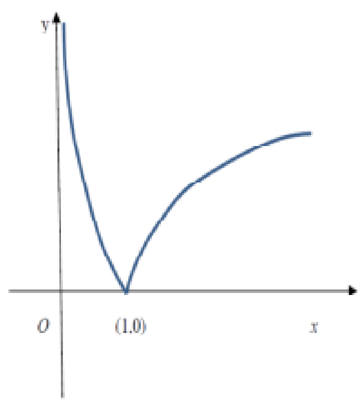
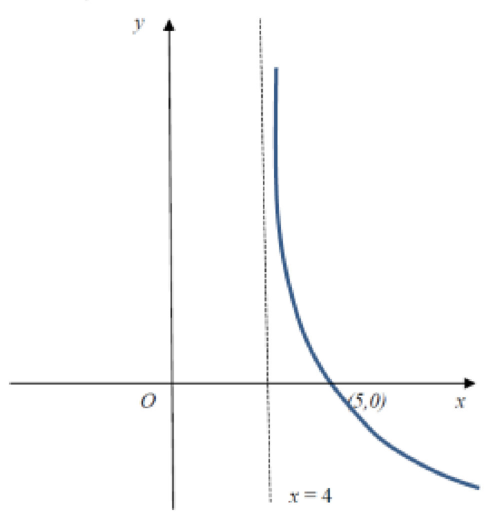
7.

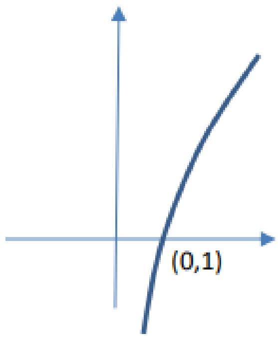
Question Number	Scheme	Marks
2(i)	 *ln* graph crossing *x* axis at (1,0) and asymptote at x=0	B1
2(ii)	 Shape including cusp	B1ft
	Touches or crosses the *x* axis at (1,0)	B1ft
	Asymptote given as x=0	B1
2(iii)	 Shape	B1
	Crosses at (5, 0)	B1ft
	Asymptote given as x=4	B1
		(7 marks)

Notes for Question 2
(i) B1 Correct shape, correct position and passing through (1, 0). Graph must 'start' to the rhs of the y - axis in quadrant 4 with a gradient that is large. The gradient then decreases as it moves through (1, 0) into quadrant 1. There must not be an obvious maximum point but condone 'slips'. Condone the point marked (0,1) on the correct axis. See practice and qualification for clarification. **Do not with hold this mark if (x=0) the asymptote is incorrect or not given**. (ii) B1ft Correct shape **including the cusp** wholly contained in quadrant 1. The shape to the rhs of the cusp should have a decreasing gradient and must not have an obvious maximum.. The shape to the lhs of the cusp should not bend backwards past (1,0) Tolerate a 'linear' looking section here but not one with incorrect curvature (See examples sheet (ii) number 3. For further clarification see practice and qualification items. Follow through on an incorrect sketch in part (i) as long as it was above and below the x axis. B1ft The curve touches or crosses the x axis at (1, 0). Allow for the curve passing through a point marked '1' on the x axis. Condone the point marked on the correct axis as (0, 1) Follow through on an incorrect intersection in part (i). B1 Award for the asymptote to the curve given/ marked as $x = 0$. Do not allow for it given/ marked as 'the y axis'. There must be a graph for this mark to be awarded, and there must be an asymptote on the graph at $x = 0$. Accept if x=0 is drawn separately to the y axis. (iii) B1 Correct shape. The gradient should always be negative and becoming less steep. It must be approximately infinite at the *lh* end and not have an obvious minimum. The lh end must not bend 'forwards' to make a C shape. The position is not important for this mark. See practice and qualification for clarification. B1ft The graph crosses (or touches) the x axis at (5, 0). Allow for the curve passing through a point marked '5' on the x axis. Condone the point marked on the correct axis as (0, 5) Follow through on an incorrect intersection in part (i). Allow for $((i)+4,0)$ B1 The asymptote is given/ marked as $x = 4$. There must be a graph for this to be awarded and there must be an asymptote on the graph (in the correct place to the rhs of the y axis). If the graphs are not labelled as (i), (ii) and (iii) mark them in the order that they are given.

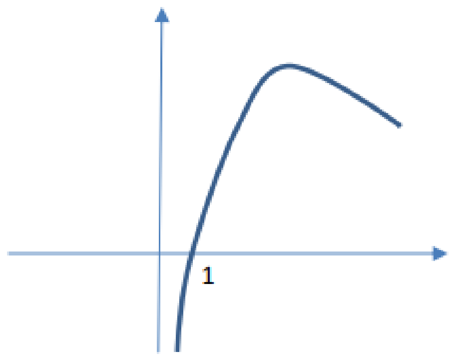
Examples of graphs in number 2

Part (i)

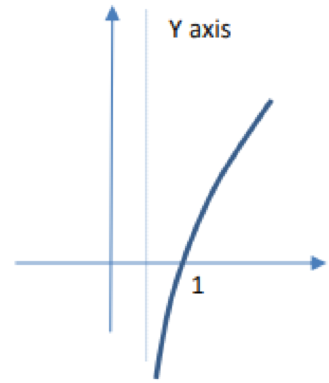
Condoned



Not condoned

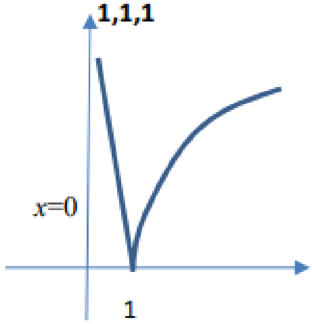


Condone

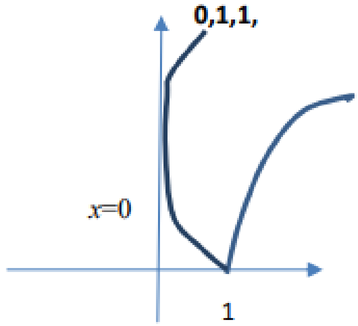


Part (ii)

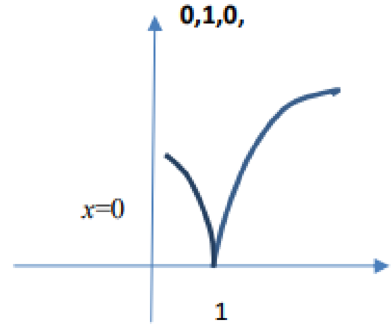
1,1,1



0,1,1,

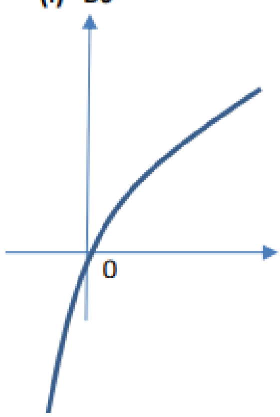


0,1,0,

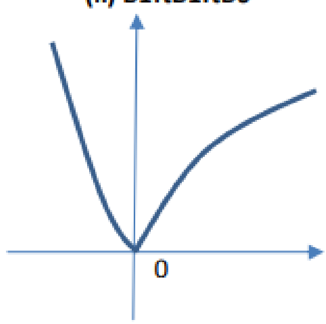


Example of follow through in part (ii) and (iii)

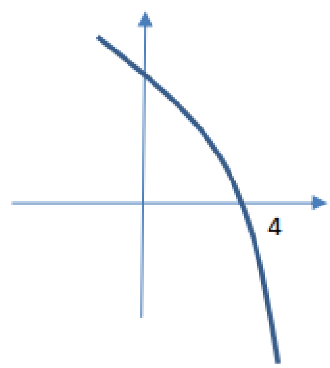
(i) B0



(ii) B1ftB1ftB0



(iii) B0B1ftB0



8.

Question Number	Scheme	Marks
7(a)	$0 \leqslant f(x) \leqslant 10$	B1
		(1)
(b)	ff(0) = f(5), = 3	B1,B1
		(2)
(c)	$y = \dfrac{4+3x}{5-x} \Rightarrow y(5-x) = 4+3x$	
	$\Rightarrow 5y - 4 = xy + 3x$	M1
	$\Rightarrow 5y - 4 = x(y+3) \Rightarrow x = \dfrac{5y-4}{y+3}$	dM1
	$g^{-1}(x) = \dfrac{5x-4}{3+x}$	A1
		(3)
(d)	$gf(x) = 16 \Rightarrow f(x) = g^{-1}(16) = 4$ oe	M1A1
	$f(x) = 4 \Rightarrow x = 6$	B1
	$f(x) = 4 \Rightarrow 5 - 2.5x = 4 \Rightarrow x = 0.4$ oe	M1A1
		(5)
		(11 marks)
Alt 1 to 7(d)	$gf(x) = 16 \Rightarrow \dfrac{4+3(ax+b)}{5-(ax+b)} = 16$	M1
	$ax + b = x - 2$ *or* $5 - 2.5x$	A1
	$\Rightarrow x = 6$	B1
	$\dfrac{4+3(5-2.5x)}{5-(5-2.5x)} = 16 \Rightarrow x = ...$	M1
	$\Rightarrow x = 0.4$ oe	A1 **(5)**

Notes for Question 7

(a)

B1 Correct range. Allow $0\leqslant f(x)\leqslant 10$, $0\leqslant f\leqslant 10$, $0\leqslant y\leqslant 10$, $0\leqslant \text{range}\leqslant 10$, $[0,10]$

Allow $f(x)\geqslant 0$ and $f(x)\leqslant 10$ but not $f(x)\geqslant 0$ or $f(x)\leqslant 10$

Do Not Allow $0\leqslant x\leqslant 10$. The inequality must include BOTH ends

(b)

B1 For correct one application of the function at $x=0$

Possible ways to score this mark are f(0)=5, f(5) $0\to 5\to ...$

B1: 3 ('3' can score both marks as long as no incorrect working is seen.)

(c)

M1 For an attempt to make x or a replaced y the subject of the formula. This can be scored for putting $y = g(x)$, multiplying across, expanding and collecting x terms on one side of the equation. Condone slips on the signs

dM1 Take out a common factor of x (or a replaced y) and divide, to make x subject of formula. Only allow **one sign error** for this mark

A1 Correct answer. No need to state the domain. Allow $g^{-1}(x)=\frac{5x-4}{3+x}$ $y=\frac{5x-4}{3+x}$

Accept alternatives such as $y=\frac{4-5x}{-3-x}$ and $y=\frac{5-\frac{4}{x}}{1+\frac{3}{x}}$

(d)

M1 Stating or implying that $f(x)=g^{-1}(16)$. For example accept $\frac{4+3f(x)}{5-f(x)}=16\Rightarrow f(x)=..$

A1 Stating $f(x)=4$ or implying that solutions are where $f(x)=4$

B1 $x=6$ and may be given if there is no working

M1 Full method to obtain other value from line $y=5-2.5x$

$5-2.5x=4\Rightarrow x=...$.

Alternatively this could be done by similar triangles. Look for $\frac{2}{5}=\frac{2-x}{4}(oe)\Rightarrow x=..$

A1 0.4 or 2/5

Alt 1 to (d)

M1 Writes $gf(x)=16$ with a linear $f(x)$. The order of gf(x) must be correct

Condone invisible brackets. Even accept if there is a modulus sign.

A1 Uses $f(x)=x-2$ or $f(x)=5-2.5x$ in the equation $gf(x)=16$

B1 $x=6$ and may be given if there is no working

M1 Attempt at solving $\frac{4+3(5-2.5x)}{5-(5-2.5x)}=16\Rightarrow x=....$. The bracketing must be correct and there must be no more than one error in their calculation

A1 $x=0.4,\frac{2}{5}$ or equivalent

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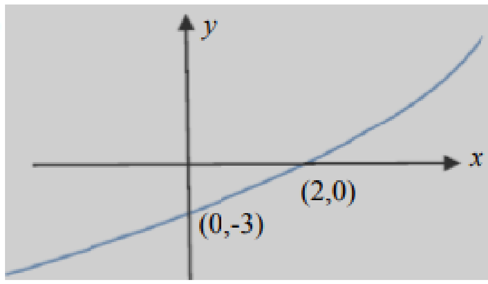
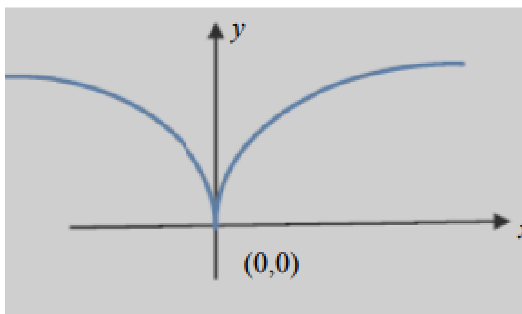
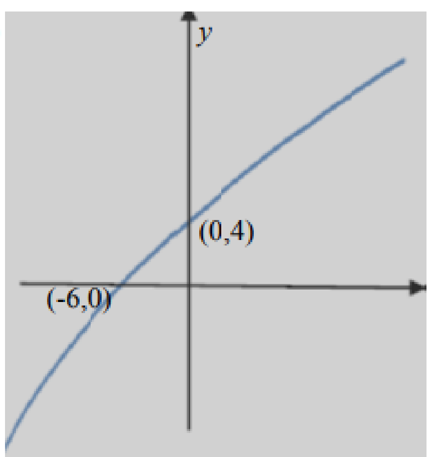
9.

Question Number	Scheme	Marks
3.	(a) ff(-3)= f(0),=2	M1,A1
		(2)
	(b)  $y = f^{-1}(x)$	
	Shape	B1
	(0,-3) and (2,0)	B1
		(2)
	(c)  y=f(|x|)-2	
	Shape	B1
	(0,0)	B1
		(2)
	(d) 	
	Shape	B1
	(-6,0) or (0,4)	B1
	(-6,0) and (0,4)	B1
		(3)
		(9 marks)

(a)	M1	A full method of finding ff(-3). f(0) is acceptable but f(-3)=0 is not. Reluctantly accept a solution obtained from two substitutions into the equation $y=\frac{2}{3}x+2$ as the line passes through both points. Do not allow for $y=\ln(x+4)$, which only passes through one of the points.
	A1	Cao ff(-3)=2. Writing down 2 on its own is enough for both marks provided no incorrect working is seen.
(b)		
	B1	For the correct shape. Award this mark for an increasing function in quadrants 3, 4 and 1 only. Do not award if the curve bends back on itself or has a clear minimum
	B1	This is independent to the first mark and for the graph passing through (0,-3) and (2, 0) Accept -3 and 2 marked on the correct axes. Accept (-3,0) and (0,2) instead of (0,-3) and (2,0) as long as they are on the correct axes Accept P'=(0,-3), Q'=(2,0) stated elsewhere as long as P'and Q' are marked in the correct place on the graph **There must be a graph for this to be awarded**
(c)		
	B1	Award for a correct shape 'roughly' symmetrical about the y- axis. It must have a cusp and a gradient that 'decreases' either side of the cusp. Do not award if the graph has a clear maximum
	B1	(0,0) lies on their graph. Accept the graph passing through the origin without seeing (0, 0) marked
(d)	B1	Shape. The position is not important. The gradient should be always positive but decreasing There should not be a clear maximum point.
	B1	The graph passes through (0,4) **or** (-6,0). See part (b) for allowed variations
	B1	The graph passes through (0,4) **and** (-6,0). See part (b) for allowed variations

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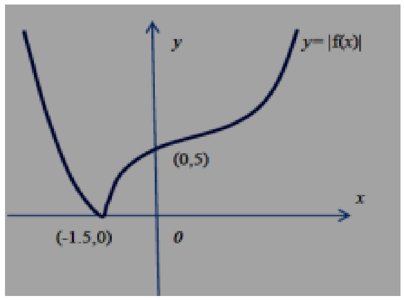
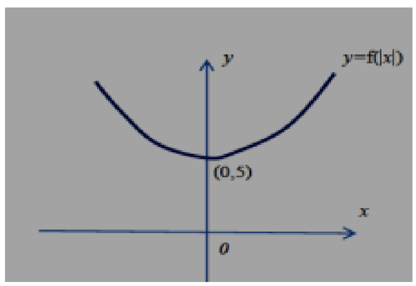
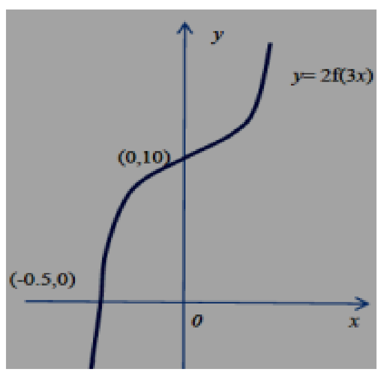
10.

Question Number	Scheme		Marks
4.(a)		Shape including cusp	B1
		(-1.5, 0) **and** (0, 5)	B1
			(2)
(b)		Shape	B1
		(0,5)	B1
			(2)
(c)		Shape	B1
		(0,10)	B1
		(-0.5, 0)	B1
			(3) **(7 marks)**

(a) **Note that this appears as M1A1 on EPEN**

B1 Shape (inc cusp) with graph in just quadrants 1 and 2. Do not be overly concerned about relative gradients, but the left hand section of the curve should not bend back beyond the cusp

B1 This is independent, and for the curve touching the x-axis at (-1.5, 0) **and** crossing the y-axis at (0,5)

(b) **Note that this appears as M1A1 on EPEN**

B1 For a U shaped curve symmetrical about the y- axis

B1 (0,5) lies on the curve

(c) **Note that this appears as M1B1B1 on EPEN**

B1 Correct shape- do not be overly concerned about relative gradients. Look for a similar shape to f(x)

B1 Curve **crosses** the y axis at (0, 10). The curve must appear in both quadrants 1 and 2

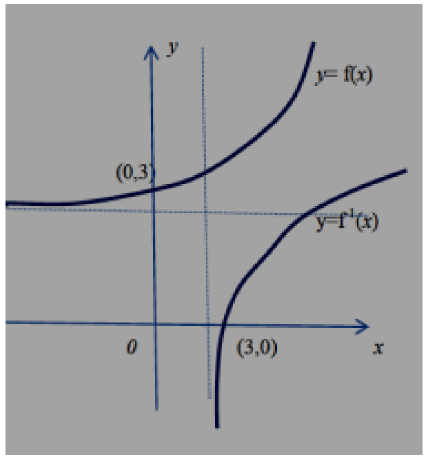
B1 Curve **crosses** the x axis at (-0.5, 0). The curve must appear in quadrants 3 and 2.

In all parts accept the following for any co-ordinate. Using (0,3) as an example, accept both (3,0) or 3 written on the y axis (as long as the curve passes through the point)

Special case with (a) and (b) completely correct but the wrong way around mark - SC(a) 0,1 SC(b) 0,1
Otherwise follow scheme

11.

Question Number	Scheme	Marks
6.	(a) $\mathrm{f}(x)>2$	B1 (1)
	(b) $\mathrm{fg}(x)=e^{\ln x}+2, =x+2$	M1,A1 (2)
	(c) $e^{2x+3}+2=6 \Rightarrow e^{2x+3}=4$	M1A1
	$\Rightarrow 2x+3=\ln 4$	
	$\Rightarrow x=\dfrac{\ln 4-3}{2}$ or $\ln 2-\dfrac{3}{2}$	M1A1 (4)
	(d) Let $y=e^x+2 \Rightarrow y-2=e^x \Rightarrow \ln(y-2)=x$	M1
	$\mathrm{f}^{-1}(x)=\ln(x-2), \quad x>2.$	A1 , **B1ft** (3)

	(e) 	Shape for f(x) (0, 3) Shape for $f^{-1}(x)$ (3, 0)	B1 B1 B1 B1 (4) **(14 marks)**

(a) B1 Range of f(x)>2. Accept y>2, (2,∞), f>2, as well as 'range is the set of numbers bigger than 2' but **don't accept** x>2

(b) M1 For applying the correct order of operations. Look for $e^{\ln x}+2$. Note that $\ln e^x+2$ is M0

A1 Simplifies $e^{\ln x}+2$ to $x+2$. Just the answer is acceptable for both marks

(c) M1 Starts with $e^{2x+3}+2=6$ and proceeds to $e^{2x+3}=...$

A1 $e^{2x+3}=4$

M1 Takes ln's both sides, $2x+3=\ln..$ and proceeds to x=….

A1 $x=\dfrac{\ln 4-3}{2}$ oe. eg $\ln 2-\dfrac{3}{2}$ Remember to isw any incorrect working after a correct answer

(d) **Note that this is marked M1A1A1 on EPEN**

M1 Starts with $y=e^x+2$ or $x=e^y+2$ and attempts to change the subject.
All ln work must be correct. The 2 must be dealt with first.
Eg. $y=e^x+2 \Rightarrow \ln y=x+\ln 2 \Rightarrow x=\ln y-\ln 2$ is M0

A1 $f^{-1}(x)=\ln(x-2)$ or $y=\ln(x-2)$ or $y=\ln|x-2|$ There must be some form of bracket

B1ft Either x>2, or follow through on their answer to part (a), provided that it wasn't $y\in\Re$
Do not accept y>2 or $f^{-1}(x)$>2.

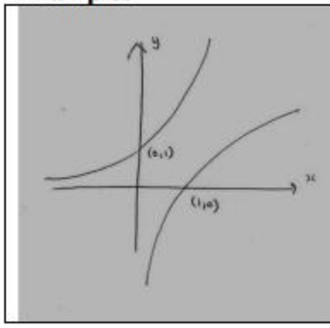
(e) B1 Shape for y=e^x. The graph should only lie in quadrants 1 and 2. It should start out with a gradient that is approx. 0 above the x axis in quadrant 2 and increase in gradient as it moves into quadrant 1. You should not see a minimum point on the graph.

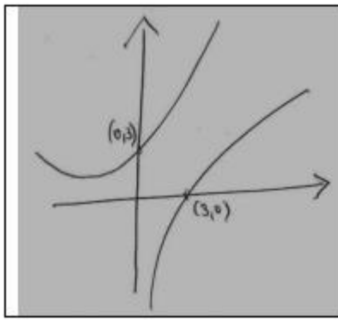
B1 (0, 3) lies on the curve. Accept 3 written on the y axis as long as the point lies on the curve

B1 Shape for y=ln$x$. The graph should only lie in quadrants 4 and 1. It should start out with gradient that is approx. infinite to the right of the y axis in quadrant 4 and decrease in gradient as it moves into quadrant 1. You should not see a maximum point. Also with hold this mark if it intersects y=e^x

B1 (3, 0) lies on the curve. Accept 3 written on the x axis as long as the point lies on the curve

Condone lack of labels in this part
Examples

	Scores **1,0,1,0.** Both shapes are fine, do not be concerned about asymptotes appearing at x=2, y=2. (See notes) Both co-ordinates are incorrect

	Scores **0,1,1,1** Shape for $y = e^x$ is incorrect, there is a minimum point on the graph. All other marks an be awarded

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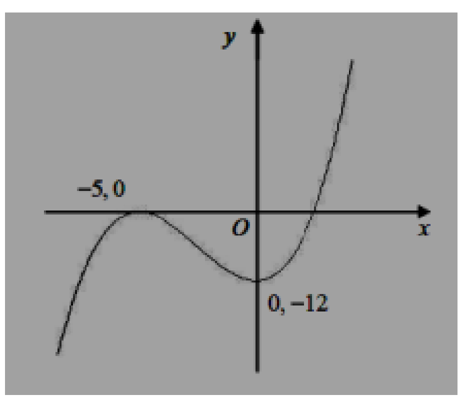
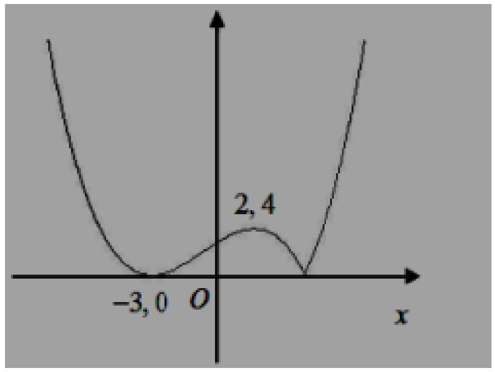
12.

Question No	Scheme		Marks
2	(a)		
		Shape	B1
		x coordinates correct	B1
		y coordinates correct	B1
			(3)

	(b)  	
	Shape	B1
	Max at (2,4)	B1
	Min at (-3,0)	B1
		(3)
		6 marks

(a)

B1 Shape unchanged. The positioning of the curve is not significant for this mark. The right hand section of the curve does not have to cross *x* axis.

B1 The x- coordinates of P' and Q' are -5 and 0 respectively. This is for translating the curve 2 units left. The minimum point Q' must be on the y axis. Accept if -5 is marked on the x axis for P' with Q' on the *y* axis (marked -12).

B1 The y- coordinates of P' and Q' are 0 and -12 respectively. This is for the stretch ×3 parallel to the *y* axis. The maximum P' must be on the *x* axis. Accept if -12 is marked on the y axis for Q' with P' on the x axis (marked -5)

(b)

B1 The curve below the x axis reflected in the x axis and the curve above the x axis is unchanged. Do not accept if the curve is clearly rounded off with a zero gradient at the x axis but allow small curvature issues. Use the same principles on the lhs- do not accept if this is a cusp.

B1 Both the x- and y- coordinates of Q', (2,4) given correctly and associated with the maximum point in the first quadrant. To gain this mark there must be a graph and it must only have one maximum. Accept as 2 and 4 marked on the correct axes or in the script as long as there is no ambiguity.

B1 Both the x- and y- coordinates of P', (-3,0) given correctly and associated with the minimum point in the second quadrant. To gain this mark there must be a graph. Tolerate two cusps if this mark has been lost earlier in the question. Accept (0, -3) marked on the correct axis.

13.

Question No	Scheme	Marks
7	(a) $2x^2+7x-4=(2x-1)(x+4)$	B1
	$\dfrac{3(x+1)}{(2x-1)(x+4)}-\dfrac{1}{(x+4)}=\dfrac{3(x+1)-(2x-1)}{(2x-1)(x+4)}$	M1
	$=\frac{x+4}{(2x-1)(x+4)}$	M1
	$=\frac{1}{2x-1}$	A1*
		(4)

	(b)	$y = \frac{1}{2x-1} \Rightarrow y(2x-1) = 1 \Rightarrow 2xy - y = 1$	
		$2xy = 1 + y \Rightarrow x = \dfrac{1+y}{2y}$	M1M1
		$y \; OR \; f^{-1}(x) = \dfrac{1+x}{2x}$	A1
			(3)
	(c)	x>0	B1
			(1)
	(d)	$\frac{1}{2\ln(x+1)-1} = \frac{1}{7}$	M1
		$\ln(x+1) = 4$	A1
		$x = e^4 - 1$	M1A1
			(4)
			12 Marks

(a)

B1 Factorises the expression $2x^2 + 7x - 4 = (2x-1)(x+4)$. This may not be on line 1

M1 Combines the two fractions to form a single fraction with a common denominator. Cubic denominators are fine for this mark. Allow slips on the numerator but one must have been adapted. Allow 'invisible' brackets. Accept two separate fractions with the same denominator. Amongst many possible options are

Correct $\frac{3(x+1)-(2x-1)}{(2x-1)(x+4)}$,Invisible bracket $\frac{3x+1-2x-1}{(2x-1)(x+4)}$,

Cubic and separate $\frac{3(x+1)(x+4)}{(2x^2+7x-4)(x+4)} - \frac{2x^2+7x-4}{(2x^2+7x-4)(x+4)}$

M1 Simplifies the (now) single fraction to one with a linear numerator divided by a quadratic factorised denominator. Any cubic denominator must have been fully factorised (check first and last terms) and cancelled with terms on a fully factorised numerator (check first and last terms).

A1* Cso. This is a given solution and it must be fully correct. All bracketing/algebra must have been correct. You can however accept $\frac{x+4}{(2x-1)(x+4)}$ going to $\frac{1}{2x-1}$ without the need for 'seeing' the cancelling

For example $\frac{3(x+1)-2x-1}{(2x-1)(x+4)} = \frac{x+4}{(2x-1)(x+4)} = \frac{1}{2x-1}$ scores B1,M1,M1,A0. Incorrect line leading to solution.

Whereas $\frac{3(x+1)-(2x-1)}{(2x-1)(x+4)} = \frac{x+4}{(2x-1)(x+4)} = \frac{1}{2x-1}$ scores B1,M1,M1,A1

(b)

M1 This is awarded for an attempt to make x or a swapped y the subject of the formula. The minimum criteria is that they start by multiplying by (2x-1) and finish with x= or swapped y=. Allow 'invisible' brackets.

M1 For applying the order of operations correctly. Allow maximum of one 'slip'. Examples of this are

$y = \frac{1}{2x-1} \rightarrow y(2x-1) = 1 \rightarrow 2x - 1 = \frac{1}{y} \rightarrow x = \frac{\frac{1}{y} \pm 1}{2}$ (allow slip on sign)

$y = \frac{1}{2x-1} \rightarrow y(2x-1) = 1 \rightarrow 2xy - y = 1 \rightarrow 2xy = 1 \pm y \rightarrow x = \frac{1 \pm y}{2y}$ (allow slip on sign)

$y = \frac{1}{2x-1} \rightarrow 2x - 1 = \frac{1}{y} \rightarrow 2x = \frac{1}{y} + 1 \rightarrow x = \frac{1}{2y} + 1$ (allow slip on ÷2)

A1 Must be written in terms of x but can be $y = \frac{1+x}{2x}$ or equivalent inc $y = \frac{\frac{1}{x}+1}{2}$, $y = \frac{x^{-1}+1}{2}$, $y = \frac{1}{2x} + \frac{1}{2}$

(c)

B1 Accept x>0, (0,∞), domain is all values more than 0. **Do not accept** x≥ 0 , y>0, [0, ∞], $f^{-1}(x) > 0$

(d)

M1 Attempt to write down fg(x) and set it equal to 1/7.
The order must be correct but accept incorrect or lack of bracketing. Eg $\frac{1}{2\ln x+1-1} = \frac{1}{7}$

A1 Achieving correctly the line $\ln(x+1) = 4$. Accept also $ln(x+1)^2 = 8$

M1 Moving from $\ln(x \pm A) = c \;\; A \neq 0 \text{ to } x =$ The ln work must be correct
Alternatively moving from $ln(x+1)^2 = c$ to $x = \cdots$
Full solutions to calculate *x* leading from $gf(x) = \frac{1}{7}$, that is $\ln\left(\frac{1}{2x-1} + 1\right) = \frac{1}{7}$ can score this mark.

A1 Correct answer only $= e^4 - 1$. Accept $e^4 - e^0$

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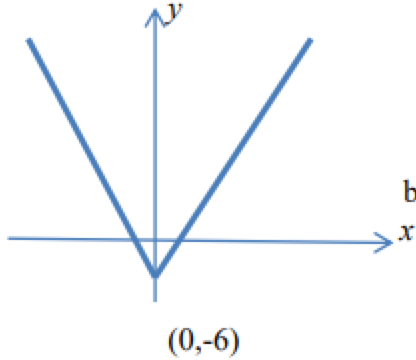
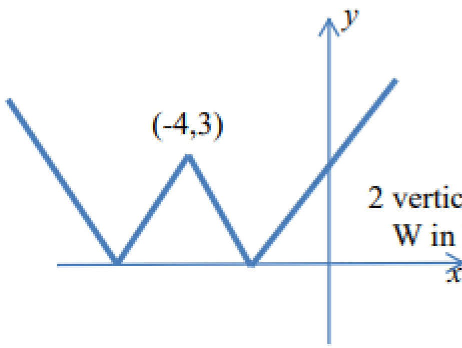
14.

Question Number	Scheme	Marks
3 (a)		
	V shape	B1
	vertex on y axis &both branches of graph cross x axis	B1
	'y' co-ordinate of R is -6	B1
		(3)
(b)		
	W shape	B1
	2 vertices on the negative x axis. W in both quad 1 & quad 2.	B1dep
	R'=(-4,3)	B1
		(3)
		6 Marks

15.

4 (a)	$y = 4 - \ln(x+2)$ $\ln(x+2) = 4 - y$ $x + 2 = e^{4-y}$ $x = e^{4-y} - 2$ $f^{-1}(x) = e^{4-x} - 2$ oe	M1 M1A1 (3)
(b)	$x \leq 4$	B1 (1)
(c)	$fg(x) = 4 - \ln(e^{x^2} - 2 + 2)$ $fg(x) = 4 - x^2$	M1 dM1A1 (3)
(d)	$fg(x) \leq 4$	B1ft (1)
		8 Marks

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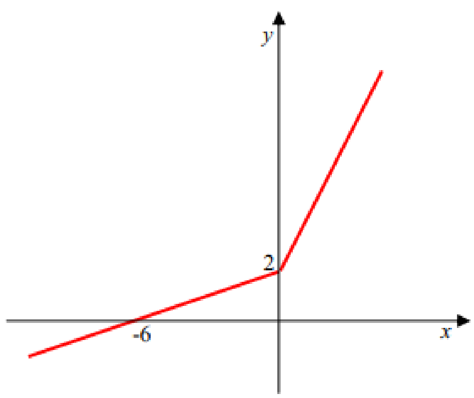
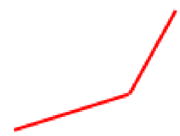
16.

Question Number	Scheme		Marks
6.			
(a)	$y = \frac{3-2x}{x-5} \Rightarrow y(x-5) = 3-2x$	Attempt to make x (or swapped y) the subject	M1
	$xy - 5y = 3 - 2x$		
	$\Rightarrow xy + 2x = 3 + 5y \Rightarrow x(y+2) = 3 + 5y$	Collect x terms together and factorise.	M1
	$\Rightarrow x = \frac{3+5y}{y+2} \quad \therefore f^{-1}(x) = \frac{3+5x}{x+2}$	$\frac{3+5x}{x+2}$	A1 oe (3)
(b)	Range of g is $-9 \leq g(x) \leq 4$ or $-9 \leq y \leq 4$	Correct Range	B1 (1)

(c)		Deduces that g(2) is 0. Seen or implied.	M1
	g g(2)= g (0) $= -6$, from sketch.	-6	A1 (2)
(d)	fg(8) = f(4)	Correct order g followed by f	M1
	$= \frac{3-4(2)}{4-5} = \frac{-5}{-1} = \underline{5}$	5	A1 (2)

Question Number	Scheme		Marks
(e)(ii)		Correct shape 	B1
		Graph goes through $(\{0\}, 2)$ and $(-6, \{0\})$ which are marked.	B1 (4)
(f)	Domain of g^{-1} is $-9 \le x \le 4$	Either correct answer or a follow through from part (b) answer	B1 $\sqrt{}$ (1) **[13]**

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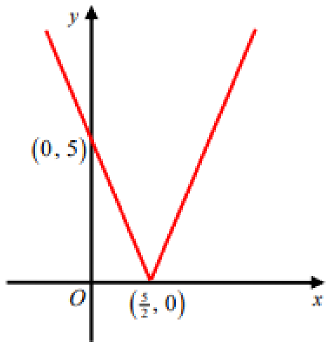
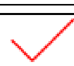
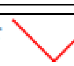
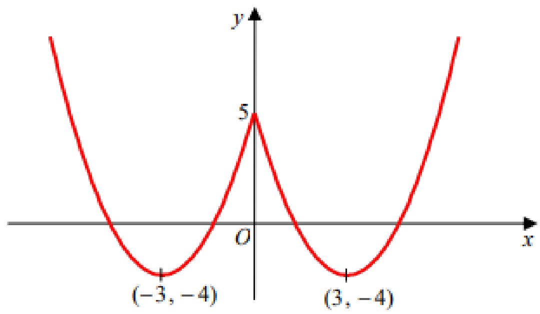
17.

Question Number	Scheme	Marks
4. (a)		M1A1 (2)
(b)	$\underline{x = 20}$ $2x-5 = -(15+x)$; $\Rightarrow \underline{x = -\frac{10}{3}}$	B1 M1;A1 oe. (3)
(c)	$\text{fg}(2) = \text{f}(-3) = \lvert 2(-3) - 5\rvert; = \lvert -11\rvert = 11$	M1;A1 (2)
(d)	$\text{g}(x) = x^2 - 4x + 1 = (x-2)^2 - 4 + 1 = (x-2)^2 - 3$. Hence $\text{g}_{\min} = -3$ Either $\text{g}_{\min} = -3$ or $\text{g}(x) \geqslant -3$ or $\text{g}(5) = 25 - 20 + 1 = 6$ $\underline{-3 \leqslant \text{g}(x) \leqslant 6}$ or $\underline{-3 \leqslant y \leqslant 6}$	M1 B1 A1 (3) **[10]**
	(a) M1: V or  or  graph with vertex on the x-axis. A1: $(\frac{5}{2}, \{0\})$ and $(\{0\}, 5)$ seen and the graph appears in both the first and second quadrants. (b) M1: Either $2x-5 = -(15+x)$ or $-(2x-5) = 15+x$ (c) M1: ***Full method*** of inserting $\text{g}(2)$ into $\text{f}(x) = \lvert 2x - 5\rvert$ or for inserting $x = 2$ into $\lvert 2(x^2 - 4x + 1) - 5\rvert$. There must be evidence of the modulus being applied. (d) M1: **Full method** to establish the minimum of g. Eg: $(x \pm \alpha)^2 + \beta$ leading to $\text{g}_{\min} = \beta$. Or for candidate to differentiate the quadratic, set the result equal to zero, find x and insert this value of x back into $\text{f}(x)$ in order to find the minimum. B1: For either finding the correct minimum value of g (can be implied by $\text{g}(x) \geqslant -3$ or $\text{g}(x) > -3$) or for stating that $\text{g}(5) = 6$. A1: $\underline{-3 \leqslant \text{g}(x) \leqslant 6}$ or $\underline{-3 \leqslant y \leqslant 6}$ or $\underline{-3 \leqslant \text{g} \leqslant 6}$. **Note that:** $-3 \leqslant x \leqslant 6$ is A0. **Note that:** $-3 \leqslant \text{f}(x) \leqslant 6$ is A0. **Note that**: $-3 \geqslant \text{g}(x) \geqslant 6$ is A0. **Note that:** $\text{g}(x) \geqslant -3$ or $\text{g}(x) > -3$ or $x \geqslant -3$ or $x > -3$ with no working gains M1B1A0. **Note that for the final Accuracy Mark**: If a candidate writes down $-3 < \text{g}(x) < 6$ or $-3 < y < 6$, then award M1B1A0. If, however, a candidate writes down $\text{g}(x) \geqslant -3$, $\text{g}(x) \leqslant 6$, then award A0. If a candidate writes down $\text{g}(x) \geqslant -3$ or $\text{g}(x) \leqslant 6$, then award A0.	

18.

Question Number	Scheme	Marks
6. (a) (i)	$(3, 4)$	B1 B1
(ii)	$(6, -8)$	B1 B1
		(4)
(b)		B1 B1 B1
		(3)
(c)	$f(x) = (x-3)^2 - 4$ or $f(x) = x^2 - 6x + 5$	M1A1
		(2)
(d)	Either: The function f is a many-one {mapping}. Or: The function f is not a one-one {mapping}.	B1
		(1)
		[10]
	(b) B1: Correct shape for $x \geqslant 0$, with the curve meeting the positive y-axis and the turning point is found below the x-axis. (providing candidate does not copy the whole of the original curve and adds nothing else to their sketch.). B1: Curve is symmetrical about the y-axis or correct shape of curve for $x < 0$. **Note:** The first two B1B1 can only be awarded if the curve has the correct shape, with a cusp on the positive y-axis and with both turning points located in the correct quadrants. Otherwise award B1B0. B1: Correct turning points of $(-3, -4)$ and $(3, -4)$. Also, $(\{0\}, 5)$ is marked where the graph cuts through the y-axis. Allow $(5, 0)$ rather than $(0, 5)$ if marked in the "correct" place on the y-axis. (c) M1: Either states $f(x)$ in the form $(x \pm \alpha)^2 \pm \beta$; $\alpha, \beta \neq 0$ Or uses a complete method on $f(x) = x^2 + ax + b$, with $f(0) = 5$ and $f(3) = -4$ to find both *a* and *b*. A1: Either $(x-3)^2 - 4$ or $x^2 - 6x + 5$ (d) B1: Or: The inverse is a one-many {mapping and not a function}. Or: Because $f(0) = 5$ and also $f(6) = 5$. Or: One y-coordinate has 2 corresponding x-coordinates {and therefore cannot have an inverse}.	

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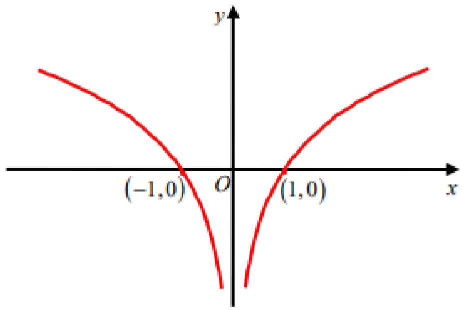
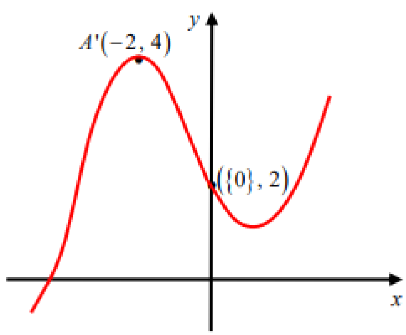
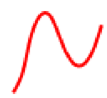
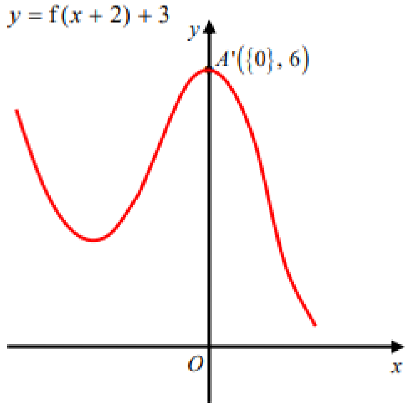
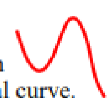
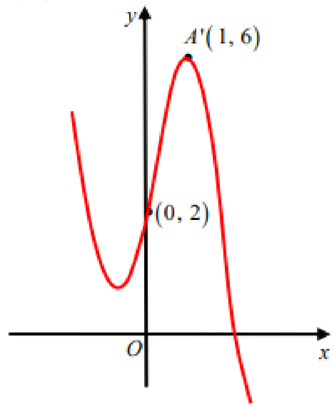
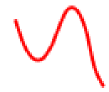
19.

Question Number	Scheme	Marks
Q5	$y = \ln\lvert x \rvert$	
		
	Right-hand branch in quadrants 4 and 1. Correct shape.	B1
	Left-hand branch in quadrants 2 and 3. Correct shape.	B1
	Completely correct sketch and both $(-1,\{0\})$ and $(1,\{0\})$	B1
		(3)
		[3]

20.

Question Number	Scheme	Marks
Q6 (i)	$y = \mathrm{f}(-x) + 1$	
		
	Shape of  and must have a maximum in quadrant 2 and a minimum in quadrant 1 or on the positive y-axis.	B1
	Either $(\{0\}, 2)$ or $A'(-2, 4)$	B1
	Both $(\{0\}, 2)$ and $A'(-2, 4)$	B1
		(3)
(ii)	$y = \mathrm{f}(x+2) + 3$	
		
	Any translation  of the original curve.	B1
	The ***translated maximum*** has either x-coordinate of 0 (can be implied) or y-coordinate of 6.	B1
	The translated curve has maximum $(\{0\}, 6)$ and is in the correct position on the Cartesian axes.	B1
		(3)

(iii)	$y = 2\mathrm{f}(2x)$ 	Shape of  with a minimum in quadrant 2 and a maximum in quadrant 1.	B1
		Either $(\{0\}, 2)$ or $A'(1, 6)$	B1
		Both $(\{0\}, 2)$ and $A'(1, 6)$	B1
			(3)
			[9]

21.

Question Number	Scheme		Marks
Q9 (i)(a)	$\ln(3x - 7) = 5$		
	$e^{\ln(3x-7)} = e^5$	Takes e of both sides of the equation. This can be implied by $3x - 7 = e^5$.	M1
	$3x - 7 = e^5 \Rightarrow x = \dfrac{e^5 + 7}{3}$ $\{= 51.804...\}$	Then rearranges to make x the subject.	dM1
		Exact answer of $\dfrac{e^5 + 7}{3}$.	A1
			(3)
(b)	$3^x e^{7x+2} = 15$		
	$\ln\left(3^x e^{7x+2}\right) = \ln 15$	Takes ln (or logs) of both sides of the equation.	M1
	$\ln 3^x + \ln e^{7x+2} = \ln 15$	Applies the addition law of logarithms.	M1
	$x\ln 3 + 7x + 2 = \ln 15$	$x\ln 3 + 7x + 2 = \ln 15$	A1 oe
	$x(\ln 3 + 7) = -2 + \ln 15$	Factorising out at least two x terms on one side and collecting number terms on the other side.	ddM1
	$x = \dfrac{-2 + \ln 15}{7 + \ln 3}$ $\{= 0.0874...\}$	***Exact answer*** of $\dfrac{-2 + \ln 15}{7 + \ln 3}$	A1 oe
			(5)

(ii) (a)	$f(x) = e^{2x} + 3, x \in \mathbb{R}$		
	$y = e^{2x} + 3 \Rightarrow y - 3 = e^{2x}$ $\Rightarrow \ln(y-3) = 2x$ $\Rightarrow \frac{1}{2}\ln(y-3) = x$	Attempt to make x (or swapped y) the subject Makes e^{2x} the subject and takes ln of both sides	M1 M1
	Hence $f^{-1}(x) = \underline{\frac{1}{2}\ln(x-3)}$	$\underline{\frac{1}{2}\ln(x-3)}$ or $\underline{\ln\sqrt{(x-3)}}$ or $\underline{f^{-1}(y) = \frac{1}{2}\ln(y-3)}$ (see appendix)	A1 cao
	$f^{-1}(x)$: Domain: $\underline{x > 3}$ or $\underline{(3, \infty)}$	Either $\underline{x > 3}$ or $\underline{(3, \infty)}$ or $\underline{\text{Domain} > 3}$.	B1 (4)
(b)	$g(x) = \ln(x-1), x \in \mathbb{R}, x > 1$		
	$fg(x) = e^{2\ln(x-1)} + 3 \quad \{= (x-1)^2 + 3\}$	An attempt to put function g into function f. $e^{2\ln(x-1)} + 3$ or $(x-1)^2 + 3$ or $x^2 - 2x + 4$.	M1 A1 isw
	$fg(x)$: Range: $\underline{y > 3}$ or $\underline{(3, \infty)}$	Either $\underline{y > 3}$ or $\underline{(3, \infty)}$ or $\underline{\text{Range} > 3}$ or $\underline{fg(x) > 3}$.	B1 (3)
			[15]